I. Introduction

Interest rates are crucial variables in measuring the strength of an economy. The reason being is that interest rates play an important role in the decision making process for both households and firms. For firms, investment decisions are strongly influenced by the interest rate. If we consider interest rates as the cost of borrowing funds, then a higher interest rate will increase the cost of borrowing which will lower investment. Households too are strongly affected by interest rates. For large purchases such as automobiles or houses, individuals must borrow funds. High interest rates will adversely affect the purchases of such large ticket items. However, high interest rates will give savers additional incentive to save.

Given the critical importance that interest rates play, we first must understand precisely what interest rates are and how they are measured.

II. Present Value

A. Calculating Present Value

Present value is the maximum amount you are willing to pay today for some amount in the future. The basic idea is that money received today is worth more than money received in the future. To illustrate this suppose that you were given a choice between receiving $100 today versus receiving $100 next year. Clearly you would want to take the $100 today. The reason why is that you could take the $100 put it in the bank and earn some interest. As a result you would have more than $100 next year.

One way you can think of present value is that it is the amount that one must put in the bank today to receive some amount in the future.

How do we calculate present value?
Let i = interest rate offered by the bank for deposits (sometimes called a simple interest rate)

Suppose that you receive $100 as a birthday gift and you want to put it in the bank which is paying a 5% interest rate. How much will you have next year?
Next year you'll get back your deposit of $100 (the principal) plus the interest payment ($5). In this case since i =5% you'll get ($100 x 0.05) = $5 in interest and thus the future amount next year will be $105.

$$\text{Future Amount Next Year} = \underset{\text{(principal)}}{\$100} + \underset{\text{(interest payment)}}{\$5}$$

We could also have re-written the formula as

Future Amount Next Year = $100 (principal) + $100i (interest payment)

Which could be re-arranged:

Future Amount Next Year = $100(1+i)
Note that you would be indifferent between choosing $100 today or $105 next year since you could take $100 today, put it in the bank and get $105 next year. Getting $100 today is equivalent to getting $105 next year. Thus the **present value** of $105 next year is $100.

In general if we let PV stand for present value (the value of money today or the amount of money you will deposit today) and FV=future value (the value of money at some future date) we can derive the formula for finding the present value of a future amount 1 year from now.

$$\boxed{PV = \frac{FV}{(1+i)}}$$

Suppose you have $100 today and interest rate is at 5%. How much will you have 2 years from now?

Amount received two years from now = FV_1 (principal next year) + $FV_1(i)$ (interest payment)

$FV_2 = FV_1\,(1 + i)$

Where FV_2 = Future amount 2 years from now
FV_1 = Future amount 1 year from now

We've already shown that $FV_1 = PV(1+ i)$

Thus FV_1 = $105 and thus FV_2 = $105(1.05) = $110.25

If $100 is deposited today we will have $110.25 2 years from now. Or we are indifferent between receiving $100 today and $110.25. The **present value** of $110.25 two years from now is $100.

Since $FV_2 = FV_1(1+ r)$ we can substitute $FV_1= PV\,(1+ r)$ to get

$FV_2 = PV\,(1+i)^2$ or $\boxed{PV = \frac{FV_2}{(1+i)^2}}$

In general to find what the present value is of some future amount n years in the future the formula will be:

$$\boxed{PV = \frac{FV_n}{(1+i)^n}}$$

This formula will be extremely important as it will allow us to calculate the value of any credit market instrument that offers payment in the future.

In-Class Example #1: Suppose you win a contest, where you will receive $10,000 today, $20,000 next year and $25,000 three years from now. What is the present value of the prize that you won? Assume that the simple interest rate is 10%.

Since some of the payments are in the future, we need to convert them to today's dollars (that is find the present value). In order to do this we need to find the present value of each of the future amounts and then sum them up.

The present value of $10,000 received today is equal to $10,000

The present value of $20,000 received next year is equal to $PV = \frac{\$20,000}{(1+0.10)^1} = \$18,181.82$

The present value of $25,000 received three years from now is equal to

$$PV = \frac{\$25,000}{(1+0.10)^3} = \$18,782.87$$

Thus the present value of the prize is equal to $10,000 + $18,181.82 + $18,782.87 =**$46,964.69**

B. Yield to Maturity

In the examples given so far, the interest rate is given. There are ways to calculate the interest rate, the most important method is called **yield-to-maturity.**

Definition: **Yield-to-maturity** is the interest rate that equates some stream of future payments from a debt instrument with its value today. In the next section, we will discuss 4 types of credit market instruments and show how the yield to maturity is calculated.

III. Types of Credit Market Instruments

A. Simple Loan

In **a simple loan,** the borrower will repay the lender a specified amount (the principal) at some date in the future (the maturity date) along with an additional payment as interest. In this debt instrument the **simple interest rate** is the interest payment divided by the loan amount (principal).

Suppose your friend wanted to borrow $1000 from you to buy some new furniture. You agree to loan the money, however you ask that the loan will be repaid next year. In addition you ask for $50 in interest.

The principal (amount of the loan) is \$1000.
The maturity date is 1 year from today
The simple interest rate is \$50/\$1000 = i = 5%

Key Point: In the case of a simple loan, the simple interest rate is also the yield to maturity.

In-Class Example #2: Horatio borrows \$900 from his uncle and next year he must pay \$940 to his uncle. What is the yield to maturity on this loan?

We know that the present value (PV) = \$900 since that is the principal.
The future value (FV) = \$940
The number of years (n) = 1

The yield to maturity is the interest rate that will equate PV and FV.

$$900 = \frac{940}{(1+i)^1}$$

We just solve for i.

\$900(1+i) = 940
(1+i) = 940/900 or [940/900] – 1 = i

i = 0.0444 or 4.44%

In-Class Exercise #3: Suppose Henrietta borrows \$1000 from her aunt and five years from today she must pay her aunt \$1100. What is the yield to maturity on this loan?

PV = \$1000
FV = \$1100
n = 5

$$1000 = \frac{1100}{(1+i)^5}$$

This problem is not as straightforward as it was in the prior example because of the denominator. We can still re-arrange the equation to get:

$\$1000(1+i)^5 = \$1100$
$(1+i)^5 = [\$1100/\$1000]$

Key is to raise both sides by the 1/5 power to eliminate the exponent on the left hand side

$(1 + i) = [\$1100/\$1000]^{1/5}$

Now you can solve as usual

$[\$1100/\$1000]^{1/5} - 1 = 1.92\%$

B. Fixed-Payment Loan

Definition: A **fixed payment loan (fully amortized loan)** is a loan where the borrower must repay the lender by making the same fixed payment every period. Each payment consists of part of the principal and part of the interest payment. Home mortgages are an example of a fixed payment loan.

The present value formula for a fixed payment loan, that is repaid over n years is the following:

$$\boxed{LV = \frac{FP}{(1+i)} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + \ldots + \frac{FP}{(1+i)^n}}$$

Where
LV = present value of the fixed payment loan (value of the loan)
FP = fixed yearly payment
n = number of years until repayment

As with the simple loan, the yield-to-maturity will be the interest rate that equates the present value formula. However, this is not easy to solve by hand. Generally, you will be given the yield to maturity.

Depending on the number of fixed payments that have to be made the calculation of the present value may become very cumbersome. We can utilize an equivalent equation for a fixed payment loan:

$$\boxed{LV = FP\left[\frac{1}{i} - \frac{1}{i(1+i)^n}\right]}$$

In-Class Exercise #4
Ms. Meyer recently took out a 20 year fixed rate home mortgage. Starting next year, she must make annual fixed payments of \$50,000. Assume that the yield-to-maturity (interest rate) is 8%. What is the amount of the loan today?

We could solve this two ways. Since we know that Ms. Meyer is going to make 20 payments of \$50,000 we can calculate the present value of each of those future payments and sum them up.

$$LV = \frac{\$50{,}000}{(1.08)} + \frac{\$50{,}000}{(1.08)^2} + \frac{\$50{,}000}{(1.08)^3} + \ldots + \frac{\$50{,}000}{(1.08)^{20}}$$

But the calculation will be long and tedious, so we can use the alternative formula for fixed payment loans:

$$LV = \$50{,}000\left[\frac{1}{0.08} - \frac{1}{0.08(1.08)^{20}}\right] = \$490{,}907.37$$

[If you were so inclined you could check this by calculating PV using the longer method, but you will find the same answer.]

We can adjust the fixed-payment formula to calculate payments that are made more often than once a year. We can easily show this using an example.

In-Class Exercise #5

Suppose that you wish to take out a 3-year loan for a Ford Focus that costs $12,451. The annual interest rate is 8% (i = 0.08). Calculate the amount of the monthly loan payment. We saw that for calculating the yearly payment for a fixed loan we use the following equation:

$$LV = FP\left[\frac{1}{i} - \frac{1}{i(1+i)^n}\right]$$

The only difference is that we are concerned about monthly payments and thus we need to calculate the monthly interest rate. A simple approximation to find the monthly interest rate is to take the annual interest rate and divide it by the number of months. Thus the monthly interest rate is 0.08/12 = 0.0067. The number of periods is n = 36 months. With this we can easily solve for the monthly fixed payment.

$$\$12{,}451 = FP\left[\frac{1}{0.0067} - \frac{1}{0.0067(1.0067)^{36}}\right]$$

FP = $390.40

C. Coupon Bond

Definition: A **coupon bond** pays the owner of the bond a fixed interest payment every year until the maturity date. At the maturity date, the face value of the bond will be repaid.

Example: A 10 year $1000 face value bond may pay $100 each (this is the fixed interest payment). In the last year, it will also pay the $1000 face value of the bond.

Every coupon bond will have the following information:
(1) Face Value of the bond (how much the bond will pay in the final payment)
(2) Issuer of the bond
(3) Maturity date (when will the face value be received)
(4) Coupon Rate. The coupon rate = (Yearly fixed coupon payment/face value) x 100%

U.S. Treasury and Corporate bonds are examples of coupon bonds.

The formula to find the present value of a coupon bond with a maturity date in n-years is:

$$\boxed{PV = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^n} + \frac{FV}{(1+i)^n}}$$

Where

PV = Present Value of the Coupon Bond (Also known as the price of the coupon bond). The reason this is also the price of the bond, is that an investor will never want to pay more than the present value of the future payment stream.
C = yearly coupon payment
FV = Face value of the coupon bond

n = number of years to the maturity date
i = yield-to-maturity

Like fixed payment loans the calculation of yield to maturity is difficult to accomplish by hand. Therefore it will always be given to you.

We could also simplify the present value equation for a coupon bond. Note that, with the exception of the face value payment at the end, the formula is similar to that of the present value of fixed payment loans. The only difference being that it is a fixed coupon payment being received rather than a fixed payment being made. We can therefore utilize a similar shortcut to derive the following equation:

$$\boxed{PV = C\left[\frac{1}{i} - \frac{1}{i(1+i)^n}\right] + \frac{FV}{(1+i)^n}}$$

In-Class Example #6

Suppose the yield to maturity is 10%. A 6 year bond with an 8% coupon rate has a face value of \$1000. Assume that the interest is paid annually. Calculate the price of this bond.

Calculate the price of the bond if the yield to maturity was 8%. Calculate the price of the bond if the yield to maturity was 12%?

We know that FV = \$1000; C = 8% x \$1000 = \$80; n = 6 and i = 10%. Plug it into the PV formula:

$$PV = \$80\left[\frac{1}{0.10} - \frac{1}{0.10(1.10)^6}\right] + \frac{\$1000}{(1.10)^6} = \$912.89$$

If i = 8%?

$$PV = \$80\left[\frac{1}{0.08} - \frac{1}{0.08(1.08)^6}\right] + \frac{\$1000}{(1.08)^6} = \$1000.00$$

If i = 12%

$$PV = \$80\left[\frac{1}{0.12} - \frac{1}{0.12(1.12)^6}\right] + \frac{\$1000}{(1.12)^6} = \$835.54$$

From In-Class Example #5 we can see some interesting facts emerge regarding coupon bonds, its price and yield to maturity.

Fact #1: When the coupon rate equals the yield-to-maturity the price of the bond will be equal to its face value.

Fact #2: When the yield-to-maturity is greater than the coupon rate, the bond price is below its face value. [We could have also shown that when the yield-to-maturity is less than the coupon rate, the bond price would be above its face value]

Fact #3: The price of a coupon bond and the yield-to-maturity are <u>negatively related.</u>

We end our discussion concerning coupon bonds by discussing a special case, called **consol bonds** (or **perpetuities).** These are coupon bonds that pay a fixed coupon payment forever. There is no maturity date, thus there is no repayment of principal.

The formula for calculating the present value of a consol bond is the same formula used to calculate the formula for a regular coupon bond. For a consol bond, n will be equal to ∞. As a result the formula simplifies to:

$$\boxed{PV = \frac{C}{i}}$$

One attractive feature of this formula is that yield-to-maturity can be easily calculated as
i = C/PV

Definition: **Current yield** is the yearly coupon payment divided by the price of the asset. For long-term bonds, the current yield is frequently used to approximate for yield-to-maturity.

D. Discount Bond

A **discount bond** (**zero-coupon bond**) is bought at a price below its face value. The face value is repaid at maturity. No interest payments are made. U.S. Treasury bills are examples of discount bonds.

The formula for finding the price of a one-year discount bond is similar to that for a simple loan.

To find the price of a one-year discount bond:

$$\boxed{PV = \frac{FV}{(1+i)}}$$

Where PV = price of the discount bond; FV = face value of discount bond; i = yield-to-maturity.

Note that for a one-year discount bond, the yield-to-maturity can be easily calculated.

$$\boxed{i = \frac{FV}{PV} - 1} \text{ or } \boxed{i = \frac{FV - PV}{PV}}$$

Note that as the price of the discount bond increases (PV), the yield-to-maturity will decrease and vice-versa.

IV. Interest Rates and Rates of Return

It is tempting to look at the yield-to-maturity as the measure of return of an asset, however that is misleading since it doesn't take into account the fact that the price of the asset can and does fluctuate over time. The rate of return takes into account price fluctuations and is thus a better measure of how well the asset is performing.

Definition: The **rate of return** of an asset is calculated as the interest payments (coupon payments) made to the owner of an asset plus any change in the value of the asset all divided by its purchase price.

Formally, to calculate the one-year rate of return for holding a bond the formula is

$$R = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

Where R = one-year return for holding a bond
C = coupon payment
P_t = purchase price of the bond
P_{t+1} = price of the bond next year

The first term (C/P_t) is just the **current yield (i_c)**. The second term $[P_{t+1} - P_t]/P_t$ is known as the **capital gain (g).**

Thus the rate of return for holding a bond one year is equal to the current yield plus the capital gain.

$$R = i_c + g$$

To show that the yield-to-maturity and the rate of return are not the same thing, consider the following example

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In-Class Exercise #7

Consider a bond with the following features:
10% coupon rate
Face Value of the bond = $1000
n = 20 years
initial yield to maturity is 10%.
initial purchase price of the bond is $1000.

Now suppose that the yield-to-maturity increases to 15%. We know that higher interest rates (yield-to-maturity) will cause the price of the bond to decline. We need to calculate the new price of the bond.

$$PV = \$100\left[\frac{1}{0.15} - \frac{1}{0.15(1.15)^{20}}\right] + \frac{\$1000}{(1.15)^{20}} = \$687.03$$

Thus we know that if the yield-to-maturity increase to 15%, the price of the bond will fall to \$687.03 next year.

Now let's find the one year rate of return from holding this bond.

We know that the coupon payment is C = \$100 and that the initial purchase price is P = \$1000. In addition, we calculated that after the rise in interest rates the bond price next year will be \$687.03

$$R = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t} = \frac{\$100}{\$1000} + \frac{\$687.03 - \$1000}{\$1000} = -21.3\%$$

The rise in the yield-to-maturity from 10% to 15% will result in a significantly different rate of return. Indeed, the rate of return is actually negative (due to the fact that the fall in bond price was greater than the return from the coupon payment).

It can be shown that prices of longer-maturity bonds will fluctuate more dramatically than shorter term bonds when interest rates changes. [You will see this clear relationship in a problem set].

Key Point: Prices and returns for longer-term bonds are more volatile than those for shorter term bonds.

The volatility of prices (and hence rate of returns) is one reason why longer-term bonds are considered more risky than shorter term bonds. The risk associated with holding longer term bonds is called **interest-rate risk**.

V. Nominal vs. Real Interest Rates

To this point, we have been discussing interest rates without considering inflation in the economy. The interest rate that is unadjusted for inflation is known as the **nominal interest rate (i)**, while the interest rate that is adjusted for inflation is known as the **real interest rate (r).** Oftentimes, the actual inflation rate is not observed so an expected rate of inflation (π^e) is used.

The real interest rate is equal to the nominal interest rate minus expected inflation.

$r = i - \pi^e$

Investors really care about real interest rates (r) as opposed to the nominal interest rate (i).

Consider the following example:

Suppose that the nominal interest rate is 5% and the expected inflation rate in the economy is 8%.

From this example we can see that the real interest rate is r = 5% - 8% = -3%. Here if you lent money, the money you will receive back next year will buy you 3% less goods and services than today because of inflation. Thus lenders will be unlikely to lend since they earn a negative real interest rate (despite the fact that the nominal interest rate is at 5%).