

# Chapter 6 Lecture Notes: Risk and Term Structure of Interest Rates

## Rates

Up to now, we discussed interest rates as if there was just a single interest rate in the economy. In reality, there are many interest rates that differ based on bond characteristics. In this chapter, we examine some theories on why some bonds have higher interest rates than others.

### I. Risk Structure of Interest Rates

Bonds may differ in interest rates even though they have the same term to maturity. **Risk structure of interest rates** helps explain why we observe interest rate differentials among otherwise similar bonds.

#### A. Default Risk

There are times when corporations run into financial difficulties and find themselves unable (or unwilling) to pay its interest payments on its debt. When these situations arise, we say that the **default risk** has increased.

Definition: **Default risk** is the risk that the bond issuer is unable to pay the face value of the bond (or is unable to make an interest payment). Bonds issued by corporations that are in financial trouble have a greater default risk than U.S. Treasury bonds. In fact, U.S. treasury notes are generally considered **default-free** since the government can always raise taxes or print money to make an interest payment. The difference in interest rate between a bond with default risk and a bond that is default free is called the **risk premium**.

Figure 1

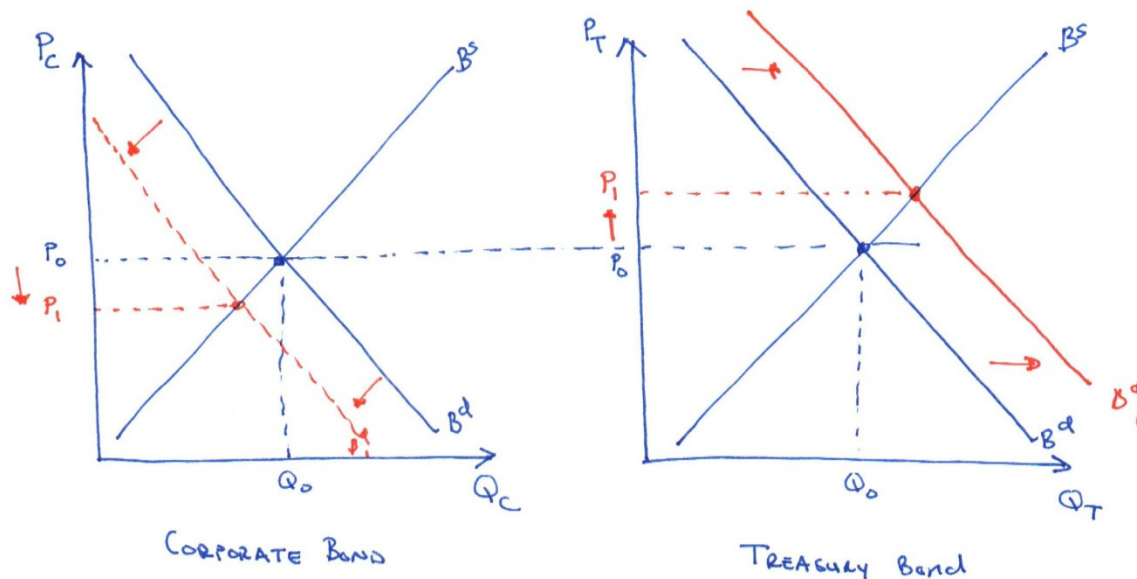


Figure 1 illustrates an example. Suppose we have a corporate and a treasury bond that initially has the same characteristics (same price, interest rate, maturity date, level of risk, etc...). Assume that something happens which makes investors question the ability of the firm to repay

its debt, the corporate bond is now considered more risky and the default risk increases. From Ch. 4, we saw that as the risk of an asset increases relative to other assets, the demand for that asset will fall. Thus the bond demand for the now riskier bond will decrease resulting in a lower price (higher interest rate). Conversely, even though the risk of treasury bonds have not changed, it is now less risky *relative* to the corporate bond. We should expect the demand for treasury bonds to increase resulting in a higher bond price (lower interest rate).

Note that the risk premium is always positive, and that as the riskiness of an asset increases, the risk premium will also increase.

Risk is measured by rating agencies, who rate bonds based on a grading system (See Table 1 in Chapter 6). The top grade is AAA. Bonds with a grade of Baa (BBB) or lower have a greater likelihood of default and are considered very speculative and are called **junk bonds (high-yield bonds)**.

During the financial crisis between 2007-2009, the trust in the credit agencies were severely tested as debt instruments that were rated AAA were unable to meet its obligations.

## B. Liquidity

In an earlier chapter it was argued that the more liquid an asset the more desirable it is. U.S. treasury bonds are the most liquid assets of all since it is widely traded. Other debt instruments are not so widely traded, and thus if one wanted to sell a bond of a small company they may find it difficult to find a buyer.

Generally, bonds that are more liquid will pay a lower interest rate than bonds that are illiquid.

Consider the following example. Suppose that there are two bonds: Bond A and Bond B. Both bonds initially have the exact same characteristics. At some point, Bond A becomes more difficult to trade and thus becomes less liquid relative to Bond B. From our study of asset demand in Chapter 4, this should decrease the demand for the illiquid bond (Bond A) and increase the demand for the relatively more liquid bond (Bond B). Graphically, this is shown again in Figure 1. The price of the less liquid bond will fall (associated with a rise in the interest rate) and the price of the more liquid bond will rise (or equivalently the interest rate will fall).

## C. Tax Treatment

One interesting fact about bonds, is that municipal bonds (bonds issued by the state and local governments) usually pay a lower interest rate than U.S. Treasury bonds. This is despite the fact that municipal bonds have a higher default risk than U.S. Treasury bonds. The reason this is the case, is due to the tax treatment of municipal bonds. Any interest earned on municipal bonds are exempt from federal taxes (and usually state taxes), while interest earned on U.S. Treasury bonds are taxable.

Suppose Merced County is offering a 1 year coupon bond paying 5% which is tax free. A 1 year U.S. Treasury note is also paying 5%, but the interest is taxed at a 20% rate. Which bond would you invest in if you had a \$1000?

How much money will you have next year if you invest in the Merced County bond?

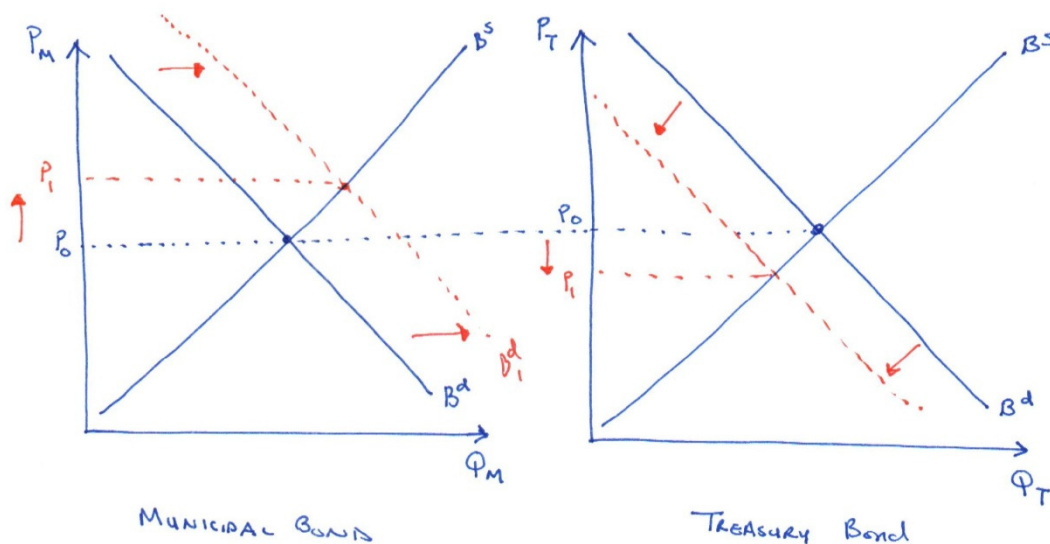
$$\$1000(1+0.05) = \$1050$$

How much money will you have next year if you invest in the U.S. Treasury Note?

$$\$1050 - \text{tax on interest} = \$1050 - \$50(0.20) = \$1040$$

Clearly you would want to invest in the Merced County bond as would every other investor. As a result the demand for Merced County bonds will increase (driving prices up and interest rates lower), while the demand for the U.S. Treasury note will decrease (driving prices down and interest rates higher).

**Figure 2**



## II. Term Structure of Interest Rates

Assume that bonds have the same risk characteristics (i.e. bonds have the same default risk, liquidity and tax treatment). In such a case bonds may still differ in interest rates because of differences in terms to maturity. A graph that plots interest rates of bonds of similar risk characteristics over varying terms to maturity are called a **yield curve**.

Three Empirical Facts about Yield Curves

**1. Interest rates on bonds of different maturities move together:** When the interest rates of short-term bonds increases, the interest rates of longer-term bonds also increase.

**2. Yield-curves can be upward sloping or downward sloping.** Depends on level of short term rates. When short-term bonds have low interest rates, the yield curves have a “normal” upward slope as in Figure 3a. However, when short-term bonds have high interest rates, the yield curves have a downward “inverted” slope. This is shown in Figure 3b

Figure 3a

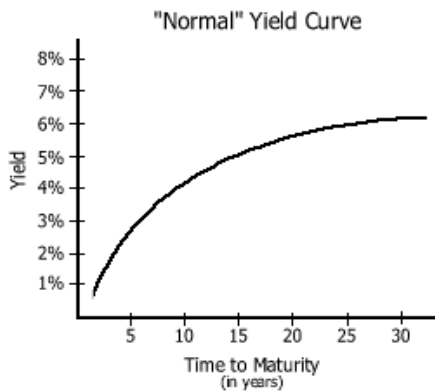
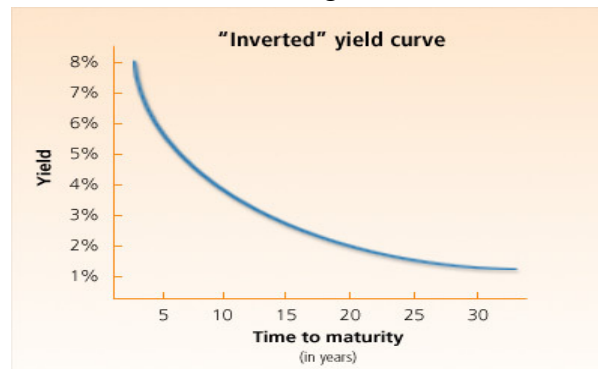


Figure 3b



### 3. Most of the time yield curves slope upwards.

Theories to Explain these Empirical Facts

- Segmented Market Theory
- Expectation Theory
- Liquidity Premium and Preferred Habitat Theories

We will discuss each of them in turn.

#### A. Segmented Market Theory

**Key Concept: Investors have a specific holding period in mind. As a result they seek only to invest in a bond that matches their investment planning horizon. In other words, bonds with different terms to maturity are not substitutes for each other.**

Example: The Jones family has a 19 year old daughter named Terri. Now Tina has recently announced that she wants to marry her co-worker Jose who is 46 years old. She tells her parents that she'll delay the wedding until she finishes her schooling in 4 years. The Jones family although upset that their daughter will marry an old man, still wants to give their daughter a lavish wedding. The Jones family went to a wedding planner who told them that a decent wedding will cost \$50,000. The family must now decide how to invest their current wealth such that they will have the \$50,000 in the future to pay for the wedding.

Suppose you are the Jones financial advisor and they come to you with three strategies of investing in the bond market. Remember they need the funds in 4 years.

**Strategy #1: Invest in a 1 year bond with YTM of 4%. Continue to reinvest in 1 year bonds thereafter.**

**Strategy #2: Invest in a 4-year bond with YTM of 5%.**

**Strategy #3: Invest in a 5-year bond with YTM of 6%. Sell the bond after four years.**

Consider Strategy #1:

In the first year the Jones family will get  $FV = P(1 + {}_0i_1)$

Where FV = Future amount

P = present value = current price of the bond

${}_0i_1$  = yield to maturity (interest rate) between years 0 and 1.

After the end of one year you take  $FV_1$  and reinvest it in another 1 year bond. The only difference is that the yield to maturity (YTM) of the one year bond between years 1 and 2 ( ${}_1i_2$ ) is not known and have to be estimated. In the second year you will get:

$$FV_2 = P(1 + {}_0i_1)(1 + {}_1i_2^e)$$

The rest of the analysis proceeds in a similar fashion. At the end of 4 years, the Jones family will receive:

$$FV_4 = P(1 + {}_0i_1)(1 + {}_1i_2^e)(1 + {}_2i_3^e)(1 + {}_3i_4^e)$$

Since the yield-to-maturity on the last three terms are not known at the beginning, there is a risk in investing in strategy #1. This is known as **re-investment risk**. If the family is risk averse, then the Jones' family should not invest in strategy #1.

Consider Strategy #2:

Strategy #2 exactly matches the investment horizon for the Jones' family. The present value formula is

$$PV = \frac{FV_4}{(1 + {}_0i_4)^4} \text{ where } {}_0i_4 = \text{YTM of the 4-year bond. This is 5\% in our example. Rearrange}$$

the terms to get:

$$FV_4 = P(1 + {}_0i_4)^4.$$

Note that all variables are known at time of the bond purchase (at  $t = 0$ ). The Jones family can obtain a certain return with no risk.

Consider Strategy #3:

Since we are going to sell the 5 year bond after 4 years, we need to calculate what the expected price of the 5-year bond will be at that time.

Use present value formula. What will the price of the bond be after 4 years? The price of any asset is just the discounted future value. Since there is only 1 period remaining on the 5 year bond, we discount the future value by the YTM between years 4 and 5.

$$P_4 = \frac{FV_5}{(1 + i_5^e)}$$
 Note that the YTM between years 4 and 5 is not known today. Which means

the price of the 5 year bond, 4 years in the future is not known today. Therefore the return on this investment is not known. This is known as **interest rate risk**. The Jones family would not want to invest in this strategy either.

Problem with the theory is that it does not explain two of the empirical facts concerning yield curves. It does not explain why interest rates on different term bonds move together, nor does it explain why yield curves are upward sloping during low short term rates (or downward sloping during high short term rates). The theory explains why yield curves are upward sloping if we assume that most investors have a short term planning horizon. If people need funds sooner rather than later, the demand for short term bonds will be greater than demand for long term bonds. Short term bonds will have higher prices and lower rates than longer term bonds.

Summary: Under Segmented Market Theory if a bond's term to maturity is less than your investment planning horizon you would not want to invest due to re-investment risk. Likewise if a bond's term to maturity is longer than your planning horizon, you would also not want to invest because of interest rate risk. **The only bonds you would demand are those whose term to maturity exactly matches your investment planning horizon.**

## B. Expectation Theory

**Key Concept: Investors are risk neutral. All investors care about is expected rate of return. If a riskier bond is offering a higher rate than another bond, the investor will always choose the risky bond because of the higher rate. In other words all bonds are perfect substitutes for each other.**

Example: Suppose we return to the Jones family again. Their daughter has informed them that she wants to drop out of college so she can marry her elderly fiancée. She tells her family that she will get married in two years at the latest. Assume that the Expectation Theory holds.

Suppose the family has two strategies this time to help pay for the wedding two years from now.

Strategy #1: Purchase a 2 year bond and hold it to maturity.

Strategy #2: Purchase a 1 year bond, and when it matures, purchase another 1 year bond.

Which strategy is better?

Step 1: Determine how much you will receive in two years employing the various strategies

Strategy #1

Strategy #2

$$P(1+i_2)^2$$

$$P(1+i_1)(1+i_2^e)$$

Where P is the initial investment.

Step 2: Consider what would happen if the amount from one of the strategies was greater than the other.

Strategy #1

Strategy #2

$$P(1+i_2)^2 < P(1+i_1)(1+i_2^e)$$

Suppose that if you chose Strategy #2 you would end up with more money in two years than if you employed Strategy #1. According to the Expectation theory any re-investment risk is disregarded because all you care about is the higher return. In the aggregate all investors will employ the same strategy, and demand more of the one year bond and less of the two year bond. As demand for the one year bond rises, prices rise and interest rate decreases. At the same time, demand for the two year bond falls causing prices to decrease and interest rates to rise. This will continue until we have equality in the returns.

**In Equilibrium:**

Strategy #1

Strategy #2

$$P(1+i_2)^2 = P(1+i_1)(1+i_2^e)$$

With this equality in hand we can solve for the two year rate in terms of the 1 yr rates.

Step 1: The P cancels from both sides of the equation.

$$(1+{}_0i_2)^2 = (1+{}_0i_1)(1+{}_1i_2^e)$$

Step 2: Use algebra to expand the terms on both sides

$$1+2{}_0i_2+{}_0i_2^2 = 1+{}_0i_1+{}_1i_2^e +({}_0i_1)({}_1i_2^e)$$

Step 3: Cancel 1 from both sides.

$$2{}_0i_2+{}_0i_2^2 = {}_0i_1+{}_1i_2^e +({}_0i_1)({}_1i_2^e)$$

Step 4: Note the following term  ${}_0i_2^2$  is likely to be very small as well as the  $({}_0i_1)({}_1i_2^e)$ . If we assume that both of these terms are approximately 0, then we can rewrite the equation as:

$$2{}_0i_2 = {}_0i_1+{}_1i_2^e$$

Step 5: Solve for  ${}_0i_2$

$${}_0i_2 = \frac{{}_0i_1+{}_1i_2^e}{2}.$$

In word the interest on a two year bond must be equal to the average rate of one year bonds.

This holds true in general:

$${}_0i_n = \frac{{}_0i_1+{}_1i_2^e + \dots + {}_{(n-1)}i_n^e}{n}.$$



The interest on a n-year bond must be equal to the average of the one year interest rate expected over n years.

### **In-Class Example #1**

How does the theory do versus the empirical facts?

- The expectations theory supports the empirical fact that interest rates tend to move together. An increase in short term rates will raise people's expectation of future short term rates. According to our formula an increase in the short term rates will increase long term rates as well.
- Theory also supports empirical fact #2. Consider when short term rates are very low. People will expect rates to rise to some normal level. Thus their expectations of future short term rates will be higher. The Expectation theory says this will raise long term rates. Long term rates will most likely be higher than short term rates. Thus the yield curve will be upward sloping. If short term rates are very high the opposite occurs, and we would expect a downward sloping yield curve.
- Cannot explain fact #3. Empirical fact is that yield curves tend to be upward sloping. However, the theory says that it is just as likely that the yield curve will be downward as upward sloping.

## C. Liquidity Premium and Preferred Habitat Theories

A third set of explanations for the variation in interest rates across bonds with different maturities is the liquidity premium theory and the preferred habitat theory. Both theories argue that investors prefer to hold short-term bonds over longer-term bonds (similar to the segmented market theory). However, unlike the segmented market theory, investors may be persuaded to hold longer-term bonds if they are compensated with higher interest.

**The Liquidity Premium Theory** states that investors tend to prefer shorter-term bonds to longer-term bonds because of lower interest rate risk. They must be compensated by a liquidity premium to hold longer term bonds.

Formally, the liquidity premium theory states that the interest rates (YTM) on a n year bond must equal the average of the expected one-year interest rates **plus** a liquidity premium.

$${}_0i_n = \frac{{}_0i_1 + {}_1i_2^e + \dots + {}_{(n-1)}i_n^e}{n} + {}_0l_n$$

Where  ${}_0l_n$  is the liquidity premium for holding a n year bond today.

The liquidity premium will always be positive and it increases as the term to maturity of the bond (n) rises.

**The Preferred Habitat Theory** assumes that investors prefer bonds of one maturity over another (preferred habitat). Investors would be only willing to buy bonds outside their preferred habitat if they are compensated with higher returns.

In-Class Example #2:

Suppose that the expected one year interest rate over the next 3 years:

$${}_0i_1 = 5\%$$

$${}_1i_2^e = 6\%$$

$${}_2i_3^e = 7\%$$

In addition assume that the liquidity premium for holding bonds of different maturities is as follows:

$${}_0l_1 = 0\% \quad = \text{liquidity preference to hold a one year bond}$$

$${}_1l_2 = 0.25\% \quad = \text{liquidity preference to hold a two year bond}$$

$${}_2l_3 = 0.5\% \quad = \text{liquidity preference to hold a three year bond}$$

- (a) Calculate the yield to maturity (interest) on a 2 year bond.
- (b) Calculate the yield to maturity (interest) on a 3 year bond.

(a) Using the formula above we have

$${}_0i_2 = \frac{5\% + 6\%}{2} + 0.25\% = 5.75\%$$

(b) Using the formula above we have

$${}_0i_3 = \frac{5\% + 6\% + 7\%}{3} + 0.50\% = 6.50\%$$

Note what happens to the interest rates as the maturity date (n) increases.

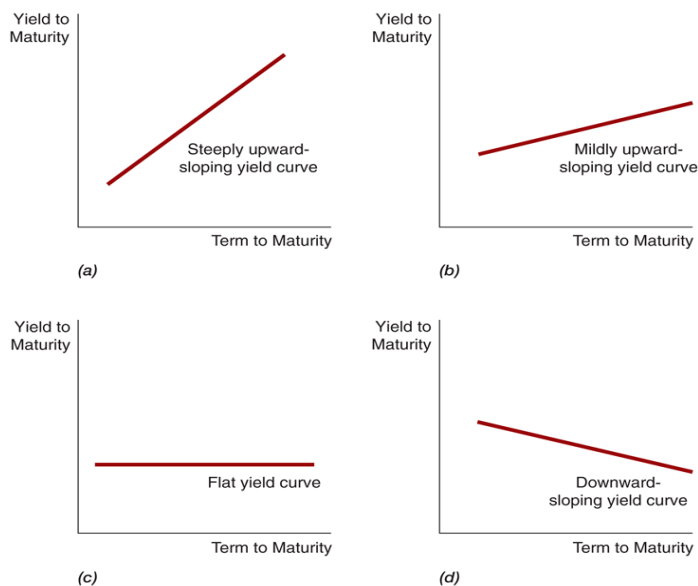
The liquidity premium and preferred habitat theories help explain all three observations regarding yield curves:

- (1) Interest rates move together over time. Although not shown, a rise in short-term interest rate will cause longer-term interest rates to also increase. [You will show this in a problem set].

(2) The two theories help explain why when short-term interest rates are low, the yield curve is upward sloping and why when short-term interest rates are high, the yield curve is downward sloping. When short-term interest rates are very low, investors expect that eventually the one year interest rates will return to normal. So they expect higher future one year interest rates. Based on the formula for calculating the interest for longer-term bonds, increases in one year interest rates will cause long-term bonds to have higher interest rates. The opposite logic would apply if the short-term interest rates are very high initially.

(3) Since both the liquidity premium and preferred habitat theories argue that people prefer short-term bonds, investors must be compensated in order to hold longer-term bonds. As a result, it will almost always be the case that longer-term bonds will offer a higher yield than a shorter term bond.

**Figure 4**



Finally, we can use our knowledge of liquidity premium and preferred habitat theories to analyze yield curves. In particular, you should be able to determine, based on the shape of the yield curve, what the market is predicting will happen to short-term interest rates.

If the yield curve is steeply upward sloping (panel a in Figure 4), then this states that longer term bonds have a much higher yield to maturity than short-term bonds. But we can see from our equation:

$${}_0i_n = \frac{{}_0i_1 + {}_1i_2^e + \dots + {}_{(n-1)}i_n^e}{n} + {}_0l_n$$

That this can only happen if future short-term interest rates are expected to rise sharply. Thus a steeply sloping yield curve tells us that investors believe that short-term interest rates will rise sharply.

If the yield curve is only slightly upward sloping (panel b) then this suggests that investors believe that future short-term interest rates will not change much.

If the yield curve is flat (panel c), then this suggests that investors believe that future short-term interest rates will fall moderately. It's expected to fall because remember that the liquidity premium is already positive.

Finally, if the yield curve is inverted, the investors believe that future short-term interest rates will fall sharply in the future.